ANALYSIS OF RESISTIVE CIRCUITS WITH CONTROLLED SOURCES

CHAPTER - 7
CONTENTS:

1. Source Conversion
2. Millman’s Theorem
3. Maximum Power Transfer Theorem
In reality, all sources—whether they are voltage or current—have some internal resistance in the relative positions shown below.

“It is important to realize that source conversions are equivalent only at their external terminals”

\[ I_L = \frac{E}{R_S + R_L} \]
\[ I_L = \frac{E}{R_S + R_L} \]

If we multiply this by a factor of 1, which we can choose to be \( R_S / R_S \)

\[ I_L = \frac{(1) E}{R_S + R_L} = \frac{(R_S / R_S) E}{R_S + R_L} = \frac{R_S (E / R_S)}{R_S + R_L} = \frac{R_S I}{R_S + R_L} \]

Same as that obtained by applying the current divider rule to the network.
EXAMPLE 1.

a. Convert the voltage source of the Figure to a current source, and calculate the current through the 4-Ω load for each source.

b. Replace the 4-Ω load with a 1-kΩ load, and calculate the current $I_L$ for the voltage source.

c. Repeat the calculation of part (b) assuming that the voltage source is ideal ($R_s = 0\Omega$) because $R_L$ is so much larger than $R_s$. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?
Solutions:

a. \[ I_L = \frac{E}{R_S + R_L} = \frac{6V}{2\Omega + 4\Omega} = 1A \]

b. \[ I_L = \frac{E}{R_S + R_L} = \frac{6V}{2\Omega + 1k\Omega} \approx 5.99A \]

c. \[ I_L = \frac{E}{R_L} = \frac{6V}{1k\Omega} = 6mA \approx 5.99mA \]

Yes, \( R_L \gg R_S \) (Voltage Source)
Millman’s Theorem

The application is that any number of parallel voltage sources can be reduced to one...
Three steps are included in its application:

**Step 1:** Convert all voltage sources to current sources

**Step 2:** Combine parallel current sources and the conductances

\[ I_T = I_1 + I_2 + I_3 \]

and

\[ G_T = G_1 + G_2 + G_3 \]
Step 3: Convert the resulting current source to a voltage source, and the desired single-source network is obtained.

**Millman’s theorem** states that for any number of parallel voltage sources,

\[ E_{eq} = \frac{\pm I_i}{G_T} = \frac{\pm I_1 \pm I_2 \pm I_3 \pm ... \pm I_N}{G_1 + G_2 + G_3 + ... + G_N} \]

or

\[ E_{eq} = \frac{\pm E_1 G_1 \pm E_2 G_2 \pm E_3 G_3 \pm ... \pm E_N G_N}{G_1 + G_2 + G_3 + ... + G_N} \]

The equivalent resistance is

\[ R_{eq} = \frac{1}{G_T} = \frac{1}{G_1 + G_2 + G_3 + ... + G_N} \]
In terms of the resistance values,

\[
E_{eq} = \pm \frac{E_1}{R_1} \pm \frac{E_2}{R_2} \pm \frac{E_3}{R_3} \pm \ldots \pm \frac{E_N}{R_N}
\]

\[
\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_N}
\]

and

\[
R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_N}}
\]
EXAMPLE 2. Using Millman’s theorem, find the current through and voltage across the resistor \( R_L \).

\[ E_{eq} = \frac{\pm \frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \]

Solution: By equation above;

\[ E_{eq} = \frac{\frac{10V}{5\Omega} - \frac{16V}{4\Omega} + \frac{8V}{2\Omega}}{\frac{1}{5\Omega} + \frac{1}{4\Omega} + \frac{1}{2\Omega}} = \frac{2A - 4A + 4A}{0.2S + 0.25 + 0.5S} \]

\[ E_{eq} = \frac{2A}{0.95S} = 2.105V \]
with 
\[ R_{eq} = \frac{1}{\frac{1}{5\Omega} + \frac{1}{4\Omega} + \frac{1}{2\Omega}} = \frac{1}{0.95\Omega} = 1.053\Omega \]

The resultant source is shown

\[ I_L = \frac{2.105V}{1.053\Omega + 3\Omega} = \frac{2.105V}{4.053\Omega} = 0.519\,A \]

with 
\[ V_L = I_L R_L = (0.519\,A)(3\Omega) = 1.557V \]
The maximum power transfer theorem states that:

A load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thévenin resistance of the network as “seen” by the load.

\[ R_L = R_{TH} \]

\[ R_L = R_N \]
\[ I = \frac{E_{TH}}{R_{TH} + R_L} \quad \text{and} \quad P_L = I^2 R_L = \left( \frac{E_{TH}}{R_{TH} + R_L} \right)^2 R_L \]

\[ P_L = \frac{E_{TH}^2 R_L}{(R_{TH} + R_L)^2} \]

Similarly,

\[ P_L = \left( \frac{I_N R_N}{R_L + R_N} \right)^2 \times R_L \]

Under maximum power conditions \((R_L = R_{Th} = R_N)\),

\[ P_{max} = \frac{E_{TH}^2}{4R_{TH}} \quad \text{and} \quad P_{max} = \frac{I_N^2 R_N}{4} \]
EXAMPLE 3. For the circuit of Figure below, a. Determine the value of load resistance required to ensure that maximum power is transferred to the load, b. Find $V_L$, $I_L$, and $P_L$ when maximum power is delivered to the load.

Solution: To determine $R_{th}$,

$$R_{th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(6k\Omega)(2k\Omega)}{6k\Omega + 2k\Omega}$$

$$R_{th} = 1.5k\Omega$$
To determine $E_{th}$,

$$V_{ab(1)} = IR_2 = \left(\frac{15V}{2k\Omega + 6k\Omega}\right)(2k\Omega) = +3.75V$$

$$V_{ab(2)} = I_2R_2 = I_T\left(\frac{R_1}{R_1 + R_2}\right)(R_2)$$

$$V_{ab(2)} = 5mA\left(\frac{6k\Omega}{6k\Omega + 2k\Omega}\right)(2k\Omega) = +7.5V$$

$$E_{TH} = V_{ab(1)} + V_{ab(2)} = +3.75V + 7.5V = 11.25V$$
Maximum power will be transferred to the load when $R_L = 1.5 \ \text{k}\Omega$.

b. Letting $R_L = 1.5 \ \text{k}\Omega$, the half of the Thévenin voltage will appear across the load resistor and half will appear across the Thévenin resistance. So, at maximum power,

$$V_L = \frac{E_{TH}}{2} = \frac{11.25V}{2} = 5.625V$$

$$I_L = \frac{V_L}{R_L} = \frac{5.625V}{1.5k\Omega} = 3.750mA$$
The power delivered to the load is found as;

\[ P_L = \frac{V_L^2}{R_L} = \frac{(5.625V)^2}{1.5k\Omega} = 21.1 \ mW \]

Or, alternatively using current, the power as

\[ P_L = I_L^2 R_L = (3.75mA)^2 (1.5k\Omega) = 21.1 \ mW \]
Thank You!